Dromion interactions of $(2+1)$-dimensional nonlinear evolution equations

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Starting from two line solitons, the solution of integrable $(2+1)$-dimensional mKdV system and KdV system in bilinear form yields a dromion solution or a ''Solitoff'' solution. Such a dromion solution is localized in all directions and the Solitoff solution decays exponentially in all directions except a preferred one for the physical field or a suitable potential. The interactions between two dromions and between the dromion and Solitoff are studied by the method of figure analysis for a $(2+1)$-dimensional modified KdV equation and a $(2+1)$-dimensional KdV type equation. Our analysis shows that the interactions between two dromions may be elastic or inelastic for different forms of solutions.

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I. INTRODUCTION

Although the soliton structures and properties of $(1+1)$-dimensional integrable nonlinear evolution equations have been now very well understood, the soliton structure in higher spatial dimensions continues to be much more intricate. Recently, since the pioneering work of Boiti *et al.* [1], the study of the exponentially localized soliton solutions called dromions and ''Solitoff'' solutions constituting an intermediate state between dromions and plane solitons in $(2+1)$-dimensional has been attracting the attention of physicists and mathematicians. Usually, dromion solutions are driven by two or more nonparallel straight-line ghost solitons. For instance, for the Davey-Stewartson (DS) [2] and the Nizhnik-Novikov-Veselov (NNV) [3] equations, their dromion solutions are driven by two perpendicular line ghost solitons [1,4]. For the Kadomtsev-Petviashvili (KP) equation, the dromion solutions are driven by nonperpendicular line ghost solitons [5]. Furthermore, there exist some dromion solution of the physical fields for one type of nonlinear models such that the DS, NNV, and asymmetrical NNV (ANNV) [6]. However, for other types of equations like the KP and the breaking soliton equations, the dromion solutions exist only for some suitable potentials of the fields [5,7]. The more generalized dromion solutions, which are driven by curved and straight line solitons for some types of $(2+1)$-dimensional nonlinear models, were found more recently [8,9,10].

In this paper, we are interested in the interactions of dromions for $(2+1)$-dimensional integrable systems. We know that soliton supplies good applied prospects in many fields of natural science such as plasmas, hydrodynamics, nonlinear optics, fiber optics, solid-state physics, and the interactive property of soliton plays an important role in developing many applications. Therefore, the studies of the interactive property of soliton for integrable models is more significant. It is well known that the interactions of $(1+1)$-dimensional solitons are elastic. This means that there is no exchange of energy (no change of shape and velocity) among interacting solitons. However, different results have been reported for $(2+1)$-dimensional integrable systems. Consequently, the dromion interactions are inelastic for the DS equation [11], but for NLBQE and the Sawada-Kotera (SK) system they are elastic [12,13]. We would like to know the reasons why the interaction between dromions is elastic for some models and inelastic for others. In addition, we also hope to learn whether there are different interactive properties when dromions are interacting due to the different selection of parameters or different form of solution for the same $(2+1)$-dimensional integrable model. To our knowledge, some works about Solitoff solution have been presented recently [14,15]. We think there may be some relationship between Solitoff solution and dromion solution. Therefore, we are also interested in this topic. In order to answer these questions, we study a $(2+1)$-dimensional integrable mKdV equation and a KdV type equation in detail.

The paper is organized as follows. In Sec. II, the multidromion solutions are given for the $(2+1)$-dimensional integrable mKdV family and KdV system. Plots of interaction between two dromions and between Solitoff and dromion for the mKdVE and KdV type equation are shown in Sec. III. Section IV includes a summary and discussion.

II. MULTIDROMION SOLUTION OF TWO $(2+1)$-DIMENSIONAL INTEGRABLE SYSTEMS

A. Multidromion solution of the mKdV family

The bilinear form of $(2+1)$-dimensional mKdV family can be written as

$$A(D_X)(f\cdot f+g\cdot g)\equiv A(D_x,D_y,D_t)(f\cdot f+g\cdot g)=0, \quad (1)$$

$$B(D_X)f\cdot g\equiv B(D_x,D_y,D_t)f\cdot g=0, \quad (2)$$

where A and B are even and odd functions of their variables $D_X=(D_x,D_y,D_t)$, $X=(x,y,t)$, respectively. The D operators are defined by [16,17]

$$D_x^n D_y^m D_t^p f\cdot g\equiv(\partial_x-\partial_{x'})^n(\partial_y-\partial_{y'})^m(\partial_t-\partial_{t'})^p \times[f(X)\cdot g(X')]|_{X'=X}. \quad (3)$$

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It may be proved that a single dromion solution of the equation system (1) and (2) exists if a physical field is defined suitably

$$w=L(\partial_X)K(\partial_X)[\tan^{-1}(g/f)]$$
$$\equiv(a_1\partial_x+b_1\partial_y)(a_2\partial_x+b_2\partial_y)[\tan^{-1}(g/f)], \tag{4}$$

where a_1, b_1, a_2, and b_2, should be selected such that the linear operators $L(\partial_X)$, $K(\partial_X)$ annihilate two line solitons

$$f=1+a_{12}\exp(\eta_1+\eta_2), \quad g=\exp(\eta_1)+\exp(\eta_2), \tag{5}$$

$$\eta_i=p_ix+q_iy+\omega_it+\text{const}\equiv\mathbf{P}_i\cdot\mathbf{X}+\text{const}, \tag{6}$$

with

$$P_i=(p_i,q_i,\omega_i), \quad (i=1,2), \quad B(P_i)=0,$$
$$a_{12}=-\frac{A(P_1-P_2)}{A(P_1+P_2)}. \tag{7}$$

That is to say, the dromion solutions are driven by ghost line solitons, which are nonparallel to each other in the space time (x,y,t). Two line solitons are annihilated by two linear operators $L(\partial_X)$ and $K(\partial_X)$ while a dromion, which is located at the cross point of the two line solitons, is survived. Performing a space transformation

$$p_1x+q_1y=px_1, \quad p_2x+q_2y=qy_1,$$
$$\Delta\equiv p_1q_2-p_2q_1\neq 0, \tag{8}$$

and fixing the constants a_i, b_i, in Eq. (4) as $a_1=-(q_1q)/\Delta$, $b_1=(p_1q)/\Delta$, $a_2=(q_2p)/\Delta$, and $b_2=-(p_2p)/\Delta$, we can rewrite Eqs. (4)–(6) as

$$w=(a_1\partial_x+b_1\partial_y)(a_2\partial_x+b_2\partial_y)[\tan^{-1}(g/f)]$$
$$\equiv\partial_{x_1}\partial_{y_1}[\tan^{-1}(g/f)], \tag{9}$$

$$f=1+a_{12}\exp(\eta_1+\eta_2), \quad g=\exp(\eta_1)+\exp(\eta_2),$$
$$\eta_1=px_1+\omega_1t+\text{const}, \quad \eta_2=qy_1+\omega_2t+\text{const}. \tag{10}$$

Now let us discuss in detail the dromion structures for the following (2+1)-dimensional integrable mKdV equation [18]:

$$u_y+u_{ttt}+u_{xxx}+3u_xv_{xx}+3u_tv_{tt}-u_x^3-u_t^3=0, \quad v_{xt}=u_xu_t. \tag{11}$$

By making use of dependent variable-related transformation

$$u=-2\tan^{-1}\left(\frac{g}{f}\right), \quad v=\log(g^2+f^2), \tag{12}$$

the bilinear forms of Eq. (11) can be shown as

$$A(D_X)(f\cdot f+g\cdot g)=D_xD_t(f\cdot f+g\cdot g)=0, \tag{13}$$

$$B(D_X)f\cdot g=(D_x^3+D_t^3+D_y)f\cdot g=0. \tag{14}$$

By means of the general method developed by Hirota, the N line soliton solution of the equation system (13) and (14) can be written as

$$f(x,y,t)=\sum_{n=0}^{N/2}\sum_{{}_NC_{2n}}a(i_1,i_2,\ldots,i_{2n})$$
$$\times\exp(\eta_{i1}+\eta_{i2}+\cdots+\eta_{i2n}), \tag{15}$$

$$g(x,y,t)=\sum_{m=0}^{[(N-1)/2]}\sum_{{}_NC_{2m+1}}a(j_1,j_2,\ldots,j_{2m+1})$$
$$\times\exp(\eta_{j1}+\eta_{j2}+\cdots+\eta_{j2m+1}), \tag{16}$$

$$a(i_1,i_2,\ldots,i_n)=\begin{cases}\Pi_{k,l}^{(n)}a(i_k,i_l) & \text{for} \quad n\geqslant 2\\ 1 \quad n=0,1,\end{cases} \tag{17}$$

$$a(i_k,i_l)=-\frac{A(P_{ik}-P_{il})}{A(P_{ik}+P_{il})}=-\frac{(p_{ik}-p_{il})(\omega_{ik}-\omega_{il})}{(p_{ik}+p_{il})(\omega_{ik}+\omega_{il})}, \tag{18}$$

$$\eta_i=p_ix+q_iy+\omega_it+\eta_{i0}, \tag{19}$$

$$B(P_{ik})=p_i^3+\omega_i^3+q_i=0. \tag{20}$$

$[N/2]$ denotes the maximum integer which does not exceed $N/2$ and n_{i0} is an arbitrary but finite real constant related to the phase of the ith soliton. ${}_NC_n$ indicates summation over all possible combination of n elements taken from N, and $\Pi_{i,l}^{(n)}$ indicates the product of all possible combinations of the n elements. From Eqs. (18) and (20), we know because a_i, b_i, are q_i, p_i dependent, the multidromion solutions for the potential w given by Eq. (9) are allowed only for a special form such that two linear operators $a_i\partial_x+b_i\,\partial_y(i=1,2)$ with fixed a_i, b_i annihilate all the line solitons. In other words, the only allowed line solitons must be perpendicular to the axes in the new space coordinates x_1 and y_1. So the multidromion solution exists only for the following potential form:

$$w=\partial_{x_1}\partial_{y_1}[\tan^{-1}g(x_1,y_1,t)/f(x_1,y_1,t)], \tag{21}$$

where the forms of $g(x_1,y_1,t)$ and $f(x_1,y_1,t)$ are the same as that of Eqs. (15) and (16), but η_i should be taken as

$$\eta_i=p_ix+q_iy+\omega_it+\eta_{i0}=p_i'x_1+\omega_it+\eta_{i0} \quad \text{or}$$
$$\eta_i=p_ix+q_iy+\omega_it+\eta_{i0}=q_i'y_1+\omega_it+\eta_{i0}. \tag{22}$$

As an example, we write down the explicit forms of f and g for $N=3$:

$$f(x,y,t)=1+a(1,2)\exp(\eta_1+\eta_2)+a(1,3)\exp(\eta_1+\eta_3)$$
$$+a(2,3)\exp(\eta_2+\eta_3), \tag{23}$$

$$g(x,y,t)=\exp(\eta_1)+\exp(\eta_2)+\exp(\eta_3)$$
$$+a(1,2)a(1,3)a(2,3)\exp(\eta_1+\eta_2+\eta_3), \tag{24}$$

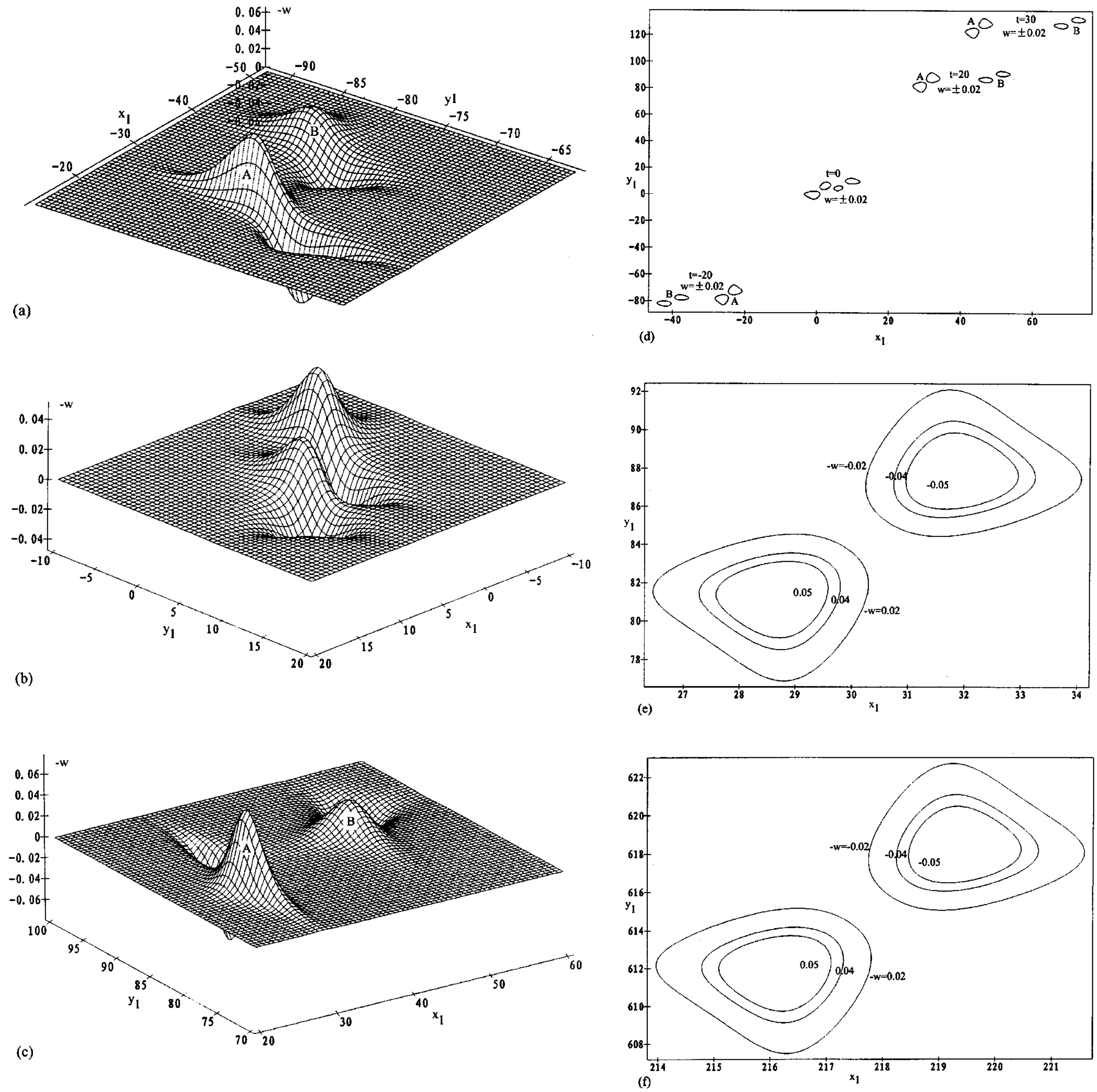

FIG. 1. The plots of the interaction of two dromions for mKdVE formed by a three-line soliton which are characterized by $\eta_1=\frac{1}{2}x+y-\frac{1}{8}9^{1/3}8^{2/3}t=\frac{1}{2}x_1-\frac{1}{8}9^{1/3}8^{2/3}t$, $\eta_2=2x+\frac{1}{2}y-\frac{1}{2}17^{1/3}2^{2/3}t=\frac{1}{2}y_1-\frac{1}{2}17^{1/3}2^{2/3}t$, and $\eta_3=x+2y-3^{1/3}t=x_1-3^{1/3}t$ about potential w. The times of the figure read: (a) $t=-20$, (b) $t=0$, (c) $t=20$. (d) Cross section plot ($w=\pm 0.02$) in correspondence with (a), (b), and (c). (e) and (f) Two cross section plots of dromion A ($w=\pm 0.02$, ± 0.04, and ± 0.05) at time taken as 20 and 150.

$$\eta_i=p_i' x+\omega_i t+\eta_{i0} \quad \text{or} \quad \eta_i=q_i' y+\omega_i t+\eta_{i0}, \tag{25}$$

$$p_i^3+\omega_i^3+q_i=0. \tag{26}$$

B. Multi-dromion solutions of the (2+1)-dimensional KdV system

The bilinear form of a $(2+1)$-dimensional KdV system is given by

$$A(D_X)f\cdot f\equiv A(D_x,D_y,D_t)f\cdot f=0, \tag{27}$$

where A is an even function of their variable. It can be proved that an n ''plane'' soliton solution, if it exists, can always be constructed for an equation of type (27) in the standard way [5,19]:

$$\begin{aligned} f=1&+\sum_{i=1}^{n}\exp(\eta_i)+\sum_{i<j}^{n}a_{ij}\exp(\eta_j+\eta_j)\\ &+\sum_{i<j<k}^{n}a_{ij}a_{ik}a_{jk}\exp(\eta_i+\eta_j+\eta_k)\\ &+\cdots+\left(\prod_{i<j}a_{ij}\right)\exp\left\{\sum_{i=1}^{n}\eta_i\right\}, \end{aligned} \tag{28}$$

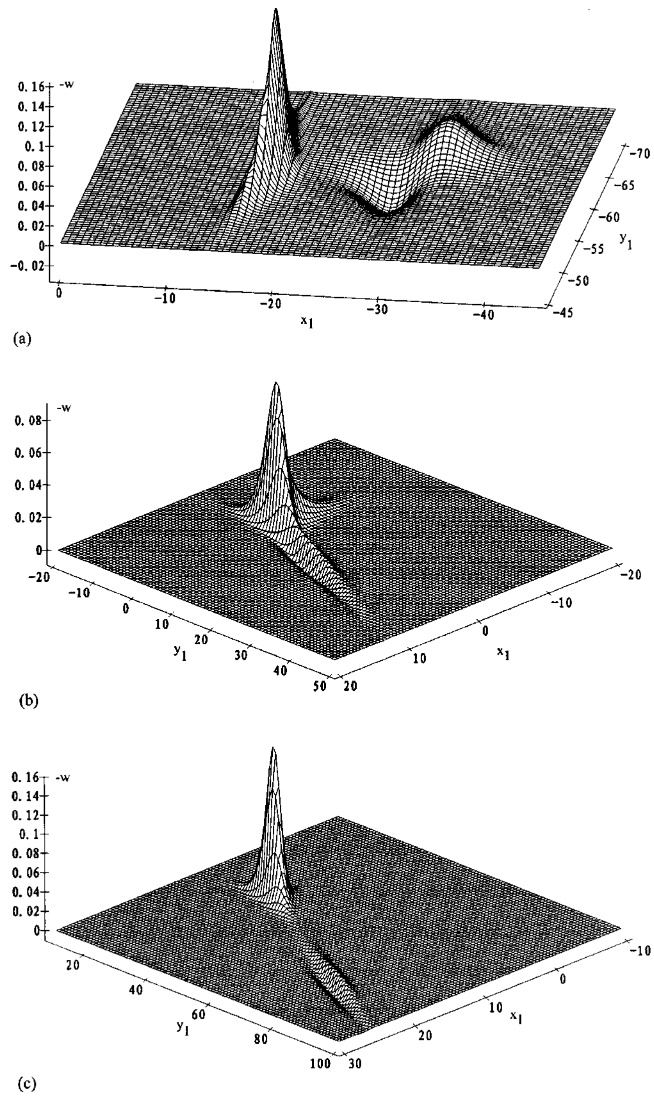

FIG. 2. The plots of the interaction of two dromions for mKdVE about potential w. The related three-line solitons are determined by $\eta_1=2x+4y-12^{1/3}t=2x_1-12^{1/3}t$, $\eta_2=2x+\frac{1}{2}y-\frac{1}{2}17^{1/3}2^{2/3}t=\frac{1}{2}y_1-\frac{1}{2}17^{1/3}2^{2/3}t$, $\eta_3=\frac{1}{2}x+y-\frac{1}{8}9^{1/3}8^{2/3}t=\frac{1}{2}x_1-\frac{1}{8}9^{1/3}8^{2/3}t=$. The times of the figure read: (a) $t=-15$, (b) $t=0$, and (c) and $t=8$.

$$a_{ij}=-\frac{A(P_i-P_j)}{A(P_i+P_j)}>0, \tag{29}$$

$$\eta_i=p_ix+q_iy+\omega_it+\eta_{i0}, \quad A(p_i,q_i,\omega_i)=0. \tag{30}$$

For the general system (28), the physical field possessing dromion solutions that are localized in all directions and constructed by line solitons, should be defined as

$$w\equiv L(\partial_X)K(\partial_X)\ln f=(a_1\partial_x+b_1\partial_y)(a_2\partial_x+b_2\partial_y)\ln f. \tag{31}$$

Equation (31) implies that two kinds of nonparallel line solitons are anihilated by two nonparallel linear operators $L(\partial_X)$ and $K(\partial_X)$ while the dromion which is located at the cross point of the two-line solitons is survived. According to the same reason above if we take transformation of Eq. (8), Eq. (31) can be changed as

$$w=\partial_{x_1}\partial_{y_1}\ln f. \tag{32}$$

For simplicity, we shall discuss the dromion structure of the following ANNV equation:

$$u_t+u_{xxx}+3\left[u\left(\int u_x dy\right)\right]_x=0 \tag{33}$$

or

$$u_t+u_{xxx}+3[uv]_x=0; \quad u_x=v_y. \tag{34}$$

The ANNV equation (34) may be considered as a model for an incompressible fluid where u and v are the components of the (dimensionless) velocity [19]. The spectral transformation for this system has been investigated in Refs. [6] and [20]. This system has been considered also in Ref. [21] as a generalization to $(2+1)$ dimensions of the results from Hirota and Satsuma [22]. The nonclassical symmetries, Painlevé property, and similarity solutions of the system have been studied by Clarkson and Mansfield [23]. Equation (33) or (34) has the bilinear form [24]

$$u=2(\log f)_{xy}, \quad v=2(\log f)_{xx}, \tag{35}$$

$$(D_yD_t+D_yD_x^3)f\cdot f=0. \tag{36}$$

Obviously, Eq. (36) has the multisolitons solution form expressed by Eq. (28). We write down the three-soliton solution expression here.

$$\begin{aligned} f=&1+\exp(\eta_1)+\exp(\eta_2)+\exp(\eta_3)+a_{12}\exp(\eta_1+\eta_2)\\ &+a_{13}\exp(\eta_1+\eta_3)+a_{23}\exp(\eta_2+\eta_3)\\ &+a_{12}a_{13}a_{23}\exp(\eta_1+\eta_2+\eta_3), \end{aligned} \tag{37}$$

$$\eta_i=p_ix+q_iy+\omega_it+\eta_{i0}, \quad q_i\omega_i+q_ip_i^3=0, \tag{38}$$

$$a_{ij}=-\frac{A(p_i-p_j)}{A(p_i+p_j)}=-\frac{(q_i-q_j)(\omega_i-\omega_j+(p_i-p_j)^3)}{(q_i+q_j)(\omega_i+\omega_j+(p_i+p_j)^3)}. \tag{39}$$

The physical field with dromion solution reads

$$u=2(\log f)_{x_1y_1}. \tag{40}$$

There is another solution form about Eq. (36) except for the standard three soliton solution of Hirota. Such solution is given by

$$\begin{aligned} f=&1+\exp(\eta_1)+\exp(\eta_2)+\exp(\eta_3)+a_{13}\exp(\eta_1+\eta_3)\\ &+a_{23}\exp(\eta_2+\eta_3), \end{aligned} \tag{41}$$

$$\begin{aligned} a_{13}&=-\frac{(q_3-q_2)(q_2+q_3-1)}{(q_2+q_3)(q_2-q_1-q_3)},\\ a_{23}&=-\frac{(q_3-q_2)(p_2-p_3)}{(p_2+p_3)(q_2+q_3)}, \quad \omega_i=-p_i^3. \end{aligned} \tag{42}$$

It can easily checked that Eq. (41) is indeed a solution of Eqs. (33) and (36).

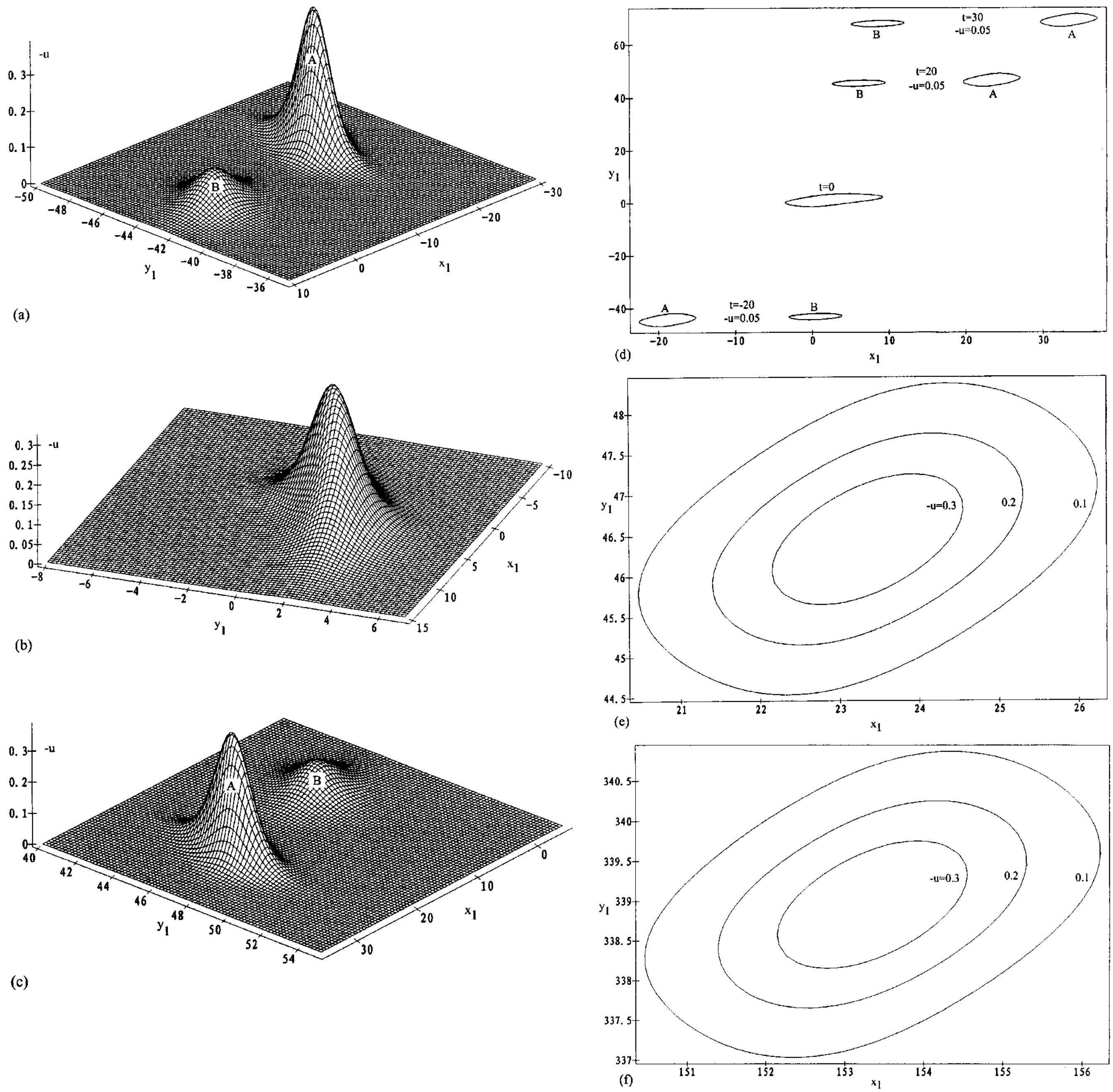

FIG. 3. The plots of the interaction of two dromions for KdVE about field u determined by Eq. (39). The solitons are characterized by $\eta_1 = x+\frac{1}{2}y-t = x_1-t$, $\eta_2=\frac{3}{2}x+\frac{3}{2}y-\frac{27}{8}t=\frac{3}{2}y_1-\frac{27}{8}t$, and $\eta_3=\frac{1}{2}x+\frac{1}{4}y-\frac{1}{8}t=\frac{1}{2}x_1-\frac{1}{8}t$. The times of the figure read: (a) $t=-20$, (b) $t=0$, and (c) $t=20$. (d) Cross section plot $(-u=0.05)$ in correspondence with (a), (b), and (c). (e) and (f) Two cross section plots of dromion A $(-u=0.1$, 0.2, and 0.3) at time taken as 20 and 150.

III. DROMION INTERACTIONS

It is known that in $(1+1)$ dimensions, there is no exchange of physical quantities like energy and momentum of the solitons after collision. Except for the phase shifts, the velocities and shapes are all remained unchanged.

We hope to know whether a similar property is valid or not for the interactions among dromions for $(2+1)$-dimensional integrable models. Especially, we hope to learn whether the interactions are dependent on parameter and solution form or not.

It is difficult to study the interaction of the dromions analytically because of the complexity of the multidromion solutions. It is more straightforward to study the dromion interactions graphically.

Figures 1 and 2 are the interaction plots of two dromions that are formed by three ghost line solitons for the mKdV equations (13) and (14) about the potential

$$w=\partial_{x_1}\partial_{y_1}\tan^{-1}\left(\frac{g}{f}\right). \tag{43}$$

In Fig. 1, the three ghost line solitons are characterized by

$$\eta_1=\tfrac{1}{2}x+y-\tfrac{1}{8}9^{1/3}8^{2/3}t=\tfrac{1}{2}x_1-\tfrac{1}{8}9^{1/3}8^{2/3}t,$$

$$\eta_2 = 2x + \tfrac{1}{2}y - \tfrac{1}{2}17^{1/3}2^{2/3}t = \tfrac{1}{2}y_1 - \tfrac{1}{2}17^{1/3}2^{2/3}t, \qquad (44)$$

$$\eta_3 = x + 2y - 3^{1/3}t = x_1 - 3^{1/3}t$$

and $a(1,2)$, $a(1,3)$, and $a(2,3)$, determined by parameter (p_i, q_i) are all nonzero in Eqs. (15) and (16). In Figs. 1(a), 1(b), and 1(c), the time t is taken as -20, 0, and 20, respectively. Figure 1(d) is a cross section plot of the two dromions before and after interaction in correspondence with Figs. 1(a), 1(b), and 1(c), respectively while $w = \text{const} = \pm 0.02$. Comparing Fig. 1(d) with Figs. 1(a)–1(c), one can clearly see that the shapes of two dromions are totally the same when they are interacting, this means there is no exchange of the energy and momentum but there are the phase shifts. Figure 1(e) and 1(f) are the cross section plots of the dromion A at time t, taken as 20 and 150, respectively, where w is taken as ± 0.02, ± 0.04, and ± 0.05. We found the shape of dromion A at time $t = 150$ is the same as that at time $t = 20$. That is to say the shape of dromion is stationary if they leave the area of interaction far away.

In Fig. 2, three ghost line solitons characterized by

$$\eta_1 = 2x + 4y - 12^{1/3}t = 2x_1 - 12^{1/3}t,$$

$$\eta_2 = 2x + \tfrac{1}{2}y - \tfrac{1}{2}17^{1/3}2^{2/3}t = \tfrac{1}{2}y_1 - \tfrac{1}{2}17^{1/3}2^{2/3}t, \qquad (45)$$

$$\eta_3 = \tfrac{1}{2}x + y - \tfrac{1}{8}9^{1/3}8^{2/3}t = \tfrac{1}{2}x_1 - \tfrac{1}{8}9^{1/3}8^{2/3}t.$$

Because of the value of the parameter in Eq. (45), $a(1,2) = 0$, $a(1,3) \neq 0$, $a(2,3) \neq 0$ in Eqs. (15) and (16). In Figs. 2(a), 2(b), and 2(c), the time is taken as -15, 0, and 8, respectively. Unlike in Fig. 1, from Figs. 2(a)–2(c), we can see that the shapes of two dromions are changed after interaction. Conclusively, there are exchanges of energy and momentum between the dromions when they are interacting.

Figure 3 is the interacting plots of two dromions, which formed by three ghost line solitons for the ANNVE (34) about physical field

$$u = 2(\log f)_{x_1 y_1}, \qquad (46)$$

where the function f is determined by Eq. (37) while three line solitons are determined by

$$\eta_1 = x + \tfrac{1}{2}y - t = x_1 - t,$$

$$\eta_2 = \tfrac{3}{2}x + \tfrac{3}{2}y - \tfrac{27}{8}t = \tfrac{3}{2}y_1 - \tfrac{27}{8}t, \qquad (47)$$

$$\eta_3 = \tfrac{1}{2}x + \tfrac{1}{4}y - \tfrac{1}{8}t = \tfrac{1}{2}x_1 - \tfrac{1}{8}t.$$

Because of the selection of parameter (p_i, q_i) in Eq. (47), the interacting constants $a(1,2)$, $a(1,3)$, and $a(2,3)$ are all nonzero in Eq. (37). In Figs. 3(a), 3(b), and 3(c), the time is taken as -20, 0, and 20, respectively. Figure 3(d) is cross section plot of the two dromions before and after interaction in correspondence with Figs. 3(a)–3(c). Figures 3(e) and 3(f) are the cross section plots of the dromion A at time taken as 20 and 150, respectively, where u is taken as -0.1, -0.2, and -0.3.

Figure 4 is also the interacting plots of two dromion of Eq. (34) about physical field (46). Function f in Eq. (46) is determined by Eq. (41) while three line solitons are characterized by

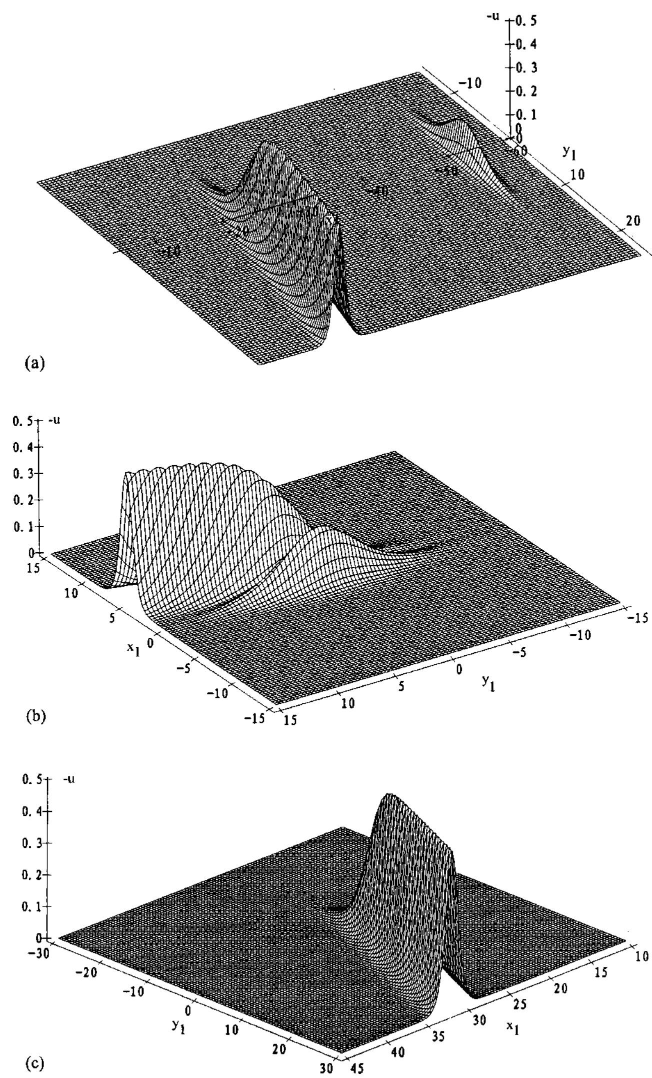

FIG. 4. A solitoff-dromion interaction of KdVE about field u determined by Eq. (44). The related three-line solitons are characterized by $\eta_1 = \frac{1}{2}y = \frac{1}{2}y_1$, $\eta_2 = 2x + y - 8t = 2x_1 - 8t$, and $\eta_3 = 3x + \frac{3}{2}y - 27t = 3x_1 - 27t$. The times in the figure are (a) $t = -6$, (b) $t = 0$, and (c) $t = 6$.

$$\eta_1 = \tfrac{1}{2}y = \tfrac{1}{2}y_1,$$

$$\eta_2 = 2x + y - 8t = 2x_1 - 8t, \qquad (48)$$

$$\eta_3 = 3x + \tfrac{3}{2}y - 27t = 3x_1 - 27t.$$

Obviously, from Fig. 3, we can see that the interacting properties between dromions for the KdV type equation (36) are the same as that of the mKdV equation shown by Fig. 1. That is to say if the solution of the equation can be taken as the standard form of Hirota, the interaction among dromions is elastic. Figure 4 show us a very interesting phenomena between Solitoff and dromion. One Solitoff and one dromion become one Solitoff after interaction.

IV. SUMMARY AND DISCUSSIONS

In summary, we have obtained some multidromion solutions of the $(2+1)$-dimensional mKdV type equation and KdV type equation for some suitable potentials or physical fields. The multidromions are constructed by multiline solitons, e.g., a single dromion is constructed by two line solitons, a two-dromion solution is constructed by three line solitons. All the line solitons should be parallel to the new axes $\{x_1, y_1\}$.

For $(1+1)$-dimensional integrable models, like the KdV equation, the interaction among solitons is completely elastic. There is no energy and momentum exchange among solitons when they are interacting. The only effect of the soliton interaction is the phase shifts. However, for the $(2+1)$-dimensional mKdV family and KdV system, there are some different interactive properties in one model because of different forms of solutions. If a multisoliton solution accords with the standard form of Hirota [all interacting constants $a(i,j)$ are nonzero], the interacting between two dromions formed by three ghost line solitons is completely elastic (there is no exchange of energy and momentum except for the phase shift). This interaction is similar to that in the $(1+1)$-dimensional models. If one of three interacting constants is zero (i.e., the form of multisoliton solution is different from the standard form of Hirota), the interaction between two dromions is not completely elastic (there is change of shape and exchange of energy and momentum). In fact, this inelastic interaction can be considered as resonant behavior of solutions [15]. We study also the behavior of solution when $t \to \infty$ through the graphical method. It is found that the shapes of dromions will be not changed when the time t increases if they leave the area of interaction far away. This means that dromions constructed in this way are stationary if they don't meet each other.

We have given out the pictures of interaction for $(2+1)$-dimensional integrable mKdVE and KdV type equation. There are different interaction behaviors between dromions in one model. The conclusion given above is similar to the collision of the classical particles. It is known that the collision between two classical particles may be elastic or inelastic according to the material properties. In this paper, we find there are the same interactive properties between two dromions, they may be elastic, or inelastic according to whether the interaction constants equal zero or not.

Though we give only the details of dromion interactions for one equation of mKdV family and one equation of KdV system here, we studied also the dromion solutions and their interactions for other types of KdV equations, like the $(2+1)$-dimensional Sawada-Kotera equation[25,13]

$$u_t = (u_{xxxx} + 5uu_{xx} + \tfrac{5}{3}u^3 + 5u_{xy})_x - 5\int u_{yy}dx + 5uu_y + 5u_x\int u_y dx \tag{49}$$

and mKdV equations, such as given by Tamizhmani *et al.* [26]

$$u_{xyt} + u_x v_{yt} + u_t v_{xy} = 0; \quad v_{xt} = u_x u_t. \tag{50}$$

Similar dromion solutions and the totally same interaction properties for Eqs. (49) and (50) can be obtained.

It seems that for us all the conclusions given above are valid for all the $(2+1)$-dimensional KdV type and mKdV type equations. Whether the similar phenomena could take place in other $(2+1)$-dimensional integrable models and/or even in $(1+1)$-dimensional integrable models is worth further studies.

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